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Scale analysis and wall–layer model for the temperature profile in a turbulent thermal convection

MYUNG KYOON CHUNG and HYO CHUL YUN
Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, 373-1, Kusong-dong, Yusung-ku, Taejon, Korea

and

RONALD J. ADRIAN
Department of Theoretical and Applied Mechanics, University of Illinois at Urbana–Champaign, Urbana, IL 61801, U.S.A.

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Abstract—Three sets of characteristic scales for the conduction layer, the transition layer and the convection layer are proposed to analyze the mean thermal structure in a turbulent thermal convection without mean motion. These scales are formulated based on molecular or turbulent eddy contribution to the momentum and heat transports in each layer. Using the proposed scales and a gradient matching technique at the interface between two adjacent layers, Kraichnan's (*Physics Fluids* **5**, 1374 (1962)) multi-layered structure of the mean temperature gradient profile is re-established. If the conduction scales are used to non-dimensionalize mean temperature gradient data near the wall, they form a plausible correlation curve that is nearly independent of the Prandtl number and the Rayleigh number for the range of experiments. From the correlation curve, it is found that the convection layer or the similarity layer with the slope of $-4/3$ begins to appear after about $z_+ \sim 15$ and the proportionality constant of the $-4/3$ power law of the mean temperature gradient is found to be about 0.6 or $d\Theta_+/dz_+ = 0.6 z_+^{-4/3}$, where Θ_+ and z_+ are non-dimensional temperature and distance scaled by the respective conduction scales. Further, a wall–layer model for the mean temperature gradient profile is formulated in accordance with the power law, $d\Theta_+/dz_+ \sim z_+^{-\alpha}$, across the layers, which is in good agreement with the data.

1. INTRODUCTION

ALTHOUGH turbulent thermal convection over a heated horizontal flat plate without mean flow has been studied by many investigators during the past decade, the mean temperature profile in the fluid layer is still controversial. The seemingly simple picture of the turbulent thermal convection is complicated by the Prandtl number effect which dictates relative magnitude between rates of momentum and heat transfer by the molecular motion. Priestley [1] in his dimensional analysis and mechanistic theory of turbulent thermal convection problem over a horizontal terrain showed that the mean temperature gradient can be represented by a power law, $dT/dz \propto z^{-\alpha}$, and suggested that $\alpha = 4/3$, the so-called similarity law. Malkus [2] applied a variational method to turbulent thermal convection in a fluid layer between two horizontal flat plates (Rayleigh convection) and predicted that $\alpha = 2$.

Later, Priestley's similarity law was theoretically supported by Kraichnan [3] who developed a modified mixing length theory for the analysis of the Rayleigh convection. His results suggest that, when the Prandtl number (Pr) of fluid is greater than a transition Prandtl number (Pr_0), at which momentum and heat are transported at the same rate, there exists a power law layer with $\alpha = 2$ between the conduction layer very close to the wall and the similarity layer. Later, Panofsky [4] again conformed the similarity layer by an application of the matching condition between the Monin–Obukov scaling and a convective scaling in the planetary boundary layer. There have been a number of experimental observations in laboratories and in the open atmosphere, however, considerable controversy on the power law profile of the mean temperature gradient still prevails.

In summary, Townsend [5], Goldstein and Chu [6], Chu and Goldstein [7] and Carrol [8] obtained power law profiles with $\alpha = 2$ in laboratory convection chambers. Croft's [9] laboratory data, data of Dyer [10] and Businger *et al.* [11] in the windless free convection region of the atmospheric boundary layer give a good correlation with $\alpha = 1.5$. Although Deardorff and Willis' [12] data for air in a range of Rayleigh numbers (Ra) between 10^6 and 10^8 follow the power law with $\alpha = 2$ more closely, they observed a tendency of the index α to approach 4/3 as Ra becomes larger; thus Priestley's similarity law seems to be an asymptotic case for $Ra \to \infty$.

The concept of a layered structure has been adopted in Kraichnan's [3] mixing length analysis and in Carrol's [8] interpretation of the thermal structure observed in a Rayleigh convection chamber with air.

NOMENCLATURE

c	coefficient (equation (32))
g	gravitational acceleration
H	distance between two plates
k	turbulent kinetic energy
Nu	Nusselt number
p	pressure fluctuation
P	mean pressure
Pr	Prandtl number
Pr_0	transition Prandtl number
Q	total kinematic heat flux
Ra	Rayleigh number
t	time
T	mean temperature
T_c	temperature of the upper plate
T_h	temperature of the bottom plate
ΔT	temperature difference between two plates
ΔT_c	temperature difference across the conduction layer
u, w	velocity fluctuations
u_τ	friction velocity
U	mean velocity
$\overline{w'_c}$	mean of r.m.s. values of the vertical advective velocity in the conduction layer
z	distance from bottom plate.

Greek symbols

α	power law index
β	thermal expansion coefficient
δ_c	conduction layer thickness
δ_{3i}	Kronecker delta
θ	temperature fluctuation
$\overline{\theta'_c}$	mean of r.m.s. temperature fluctuations in the conduction layer
Θ_+	non-dimensionalized temperature
κ	molecular diffusivity of heat, or von Karman constant
κ_b	buoyancy similarity constant
κ_t	eddy diffusivity of heat
ν	molecular viscosity
ν_t	eddy diffusivity of momentum
ρ	density
τ_w	wall shear stress.

Subscripts

c	conduction layer
i, j	Cartesian tensor indices, or reference layer indices
κ	transition layer for $Pr \ll Pr_0$
ν	transition layer for $Pr \gg Pr_0$
$*$	convection layer
$+$	dimensionless value by conduction scale.

Superscript

$\sim$	instantaneous variables.

Carrol's results did not, however, conform Kraichnan's three-layered structure. Instead, he argued that it consists of a 'conduction layer' very close to the bottom plate, a 'transition layer' in which $\alpha = 2$ and an 'interior region' in which $\alpha \to \infty$.

Recently, there has been considerable progress in developing computational turbulence models for predicting buoyancy-affected convection flows; for example, Zeman and Lumley [13], Ljuboja and Rodi [14], Chung and Sung [15], Shih and Lumley [16] and Weinstock [17]. Since, however, most computational models are valid for high Peclet number flows, a wall function which describes the mean temperature variation near the wall is definitely called for in order to provide the near-wall boundary conditions for the high-Peclet-number-modeled turbulence equations which work only within the interior fluid layer. A similar approach has been widely used in computing wall-bounded shear flows by employing the well-known logarithmic mean velocity profile scaled by the plausible characteristic friction velocity $u_\tau = \sqrt{(\tau_w/\rho)}$ and length ν/u_τ. In order to find such a wall function for the mean temperature profile in the turbulent thermal convection, it is required to clarify further the controversial power law and to devise proper characteristic wall scales of velocity, length and temperature. These requirements are the motivation of the present study.

2. CHARACTERISTIC SCALES

A new four-layered structure is proposed as in Fig. 1, where most responsible mechanisms for transport of momentum and heat are shown for two cases $Pr \gg Pr_0$ and $Pr \ll Pr_0$, separately. Recall that the transition Prandtl number Pr_0 was estimated to be about 0.1 by Kraichnan [3].

Evidently, transport processes in layer I, the 'conduction layer', are dominated by molecular viscosity (ν) and molecular diffusivity of heat (κ). In layers III and IV, the 'convection layer' and the 'buoyancy-defect layer' [18], respectively, eddy diffusivity of momentum (ν_t) and heat (κ_t) are assumed to be predominant. For fluid with $Pr \gg Pr_0$, molecular viscosity and eddy diffusivity are important mechanisms in the 'transition layer' (layer II), and the converse is true for fluid with $Pr \ll Pr_0$. Relevant governing equations for the flow field without mean motion are

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j - \overline{u_i u_j}) = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\nu}{w_p z_p}\frac{\partial^2 u_i}{\partial x_j^2} + \frac{\theta_p z_p}{w_p^2} g\beta\theta\delta_{3i} \quad (1)$$

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j}\left(\frac{\kappa}{w_p z_p}\frac{\partial T}{\partial x_j} - \overline{u_j \theta}\right) \quad (2)$$

where the Boussinesq approximation and Reynolds

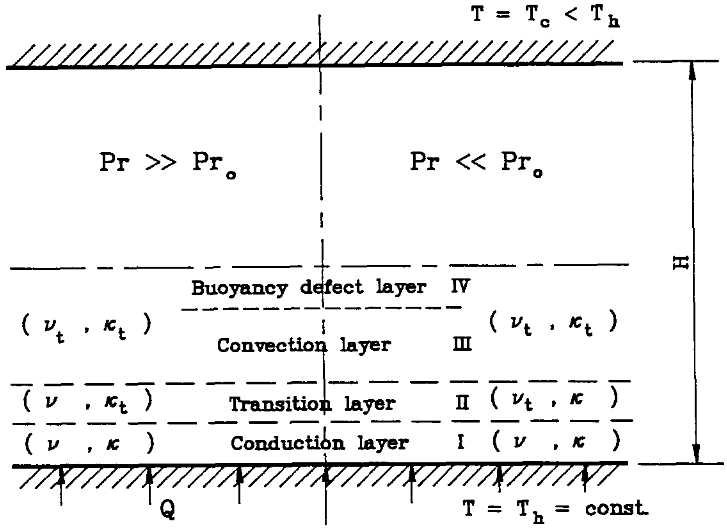

FIG. 1. A proposed four-layered structure and transport mechanisms in turbulent thermal convection.

decomposition, $\tilde{u}_i = U_i + u_i$, $\tilde{T} = T + \theta$, $\tilde{P} = P + p$ have been used and nondimensionalized by a velocity scale w_p, a length scale z_p and a temperature defect scale θ_p. For a steady flow field, the total kinematic heat flux Q which is the sum of the molecular and the turbulent kinematic heat fluxes

$$Q = \alpha \frac{dT}{dz} + \overline{w\theta}$$

is constant across the layers, and it is an important independent reference quantity which relates θ_p and w_p as

$$Q = w_p \theta_p \tag{3}$$

throughout the layers. The buoyancy term in equation (1) is most active at any point in the fluid which causes the fluid to be in turbulent convective motion. This observation and relation (3) lead us to another scaling law

$$\frac{g\beta Q z_p}{w_p^3} = 1. \tag{4}$$

(a) *Convection layer III*

Now, since molecular terms in equations (1) and (2) are negligible in layer III, the characteristic length scale of large-scale eddy motion should be determined by the geometry. Hence, we choose $z_p = H$, the distance between two plates. Denoting the characteristic scales in layer III by a subscript $_*$, equations (3) and (4) and the length scale, $z_p = H$, yield the following scaling laws:

$$z_* = H \tag{5}$$

$$w_* = (g\beta Q z_*)^{1/3} \tag{6}$$

$$\theta_* = Q/w_* \tag{7}$$

which are precisely the same as Deardorff [19].

(b) *Transition layer II*

When $Pr \gg Pr_0$, the diffusion term in equation (2) is negligible but that in equation (1) must be set to a constant value.

For convenience, we let

$$\frac{\nu}{w_p z_p} = 1. \tag{8}$$

Then equations (3), (4) and (8) determine the following transition scaling law for $Pr \gg Pr_0$:

$$w_\nu = (g\beta\nu Q)^{1/4} \tag{9}$$

$$z_\nu = \frac{\nu}{w_\nu} \tag{10}$$

$$\theta_\nu = \frac{Q}{w_\nu}. \tag{11}$$

Similarly, for $Pr \ll Pr_0$, we obtain

$$w_\kappa = (g\beta\kappa Q)^{1/4} \tag{12}$$

$$z_\kappa = \frac{\kappa}{w_\kappa} \tag{13}$$

$$\theta_\kappa = \frac{Q}{w_\kappa}. \tag{14}$$

(c) *Conduction layer I*

In the conduction layer, both the momentum and heat are transferred by the molecular motions, and Long [18] assumed that the advective motion in the vertical direction is balanced by the conduction

$$\overline{w'_c}\frac{\delta_c}{\kappa} \sim 1$$

where w'_c is the mean of r.m.s values of the vertical advective velocity in the conduction layer and δ_c the conduction layer thickness. Also it is assumed by Long [18] that within the conduction layer, the buoyancy force, $g\beta\overline{\theta'_c}$, where $\overline{\theta'_c}$ is the mean of r.m.s. temperature fluctuations, and the viscous force, $\nu\overline{w'_c}/\delta_c^2$, are of the same order

$$g\beta\overline{\theta'_c} \sim \nu\frac{\overline{w'_c}}{\delta_c^2}.$$

But, since most heat is transferred by the molecular conduction in the conduction layer, we may have the following order-of-magnitude relation:

$$Q \sim \kappa\Delta T_c/\delta_c$$

where ΔT_c is the temperature drop across the conduction layer. Moreover, since $\overline{\theta'_c}$ is of the order of ΔT_c, the following approximations can be made:

$$Q \sim \kappa\overline{\theta'_c}/\delta_c \sim \kappa\nu\overline{w'_c}/(g\beta\delta_c^3) \sim \kappa\nu\overline{w'_c}^4/(g\beta\kappa^3).$$

Consequently, if such an approximate value of $\overline{w'_c}$ in the above relation is selected as the characteristic velocity scale in the conduction layer, we finally have

$$w_c = \left(\frac{\kappa^2}{\nu}g\beta Q\right)^{1/4}. \tag{15}$$

Then, the length scale z_c can be set as the same as $\delta_c \sim \kappa/\overline{w'_c}$

$$z_c = \frac{\kappa}{w_c} \tag{16}$$

and the temperature scale is estimated by equation (3) as follows:

$$\theta_c = \frac{Q}{w_c}. \tag{17}$$

3. GRADIENT MATCHING AND POWER LAWS

Let us assume that the mean temperature profiles in two adjacent layers, i and j, can be scaled by

$$\frac{T-T_i}{\theta_i} = f_i\left(\frac{z}{z_i}\right) \tag{18}$$

and

$$\frac{T-T_j}{\theta_j} = f_j\left(\frac{z}{z_j}\right) \tag{19}$$

where T_i and T_j are reference temperatures, z_i and z_j are length scales and θ_i and θ_j are temperature defect scales for layers i and j, respectively. Then, a smooth gradient matching at the interface between the two layers requires the following condition:

$$\frac{df_i(z/z_i)}{d(z/z_i)} = \left(\frac{z_i}{z_j}\right)\left(\frac{\theta_j}{\theta_i}\right)\frac{df_j(z/z_j)}{d(z/z_j)}. \tag{20}$$

(a) *Case* 1, $Pr \gg Pr_0$

When we apply condition (20) to the interface between the conduction layer and the transition layer, the scale relations (11) and (17) make the coefficient in the right-hand side of equation (20) to be $(z_c/z_v)(\theta_v/\theta_c) = (z_c/z_v)\times(w_c/w_v)$. Using equations (9) and (15), it is easy to show that $w_c/w_v = z_c/z_v$. Then, the coefficient becomes $(z_c/z_v)^2$. Finally, multiplying both sides by $(z/z_c)^2$ yields the following equation:

$$\left(\frac{z}{z_c}\right)^2\frac{df_c(z/z_c)}{d(z/z_c)} = \left(\frac{z}{z_v}\right)^2\frac{df_v(z/z_v)}{d(z/z_v)}. \tag{21}$$

Since the left-hand side of this equation depends only on z/z_c and the right-hand side only on z/z_v, each side must be a constant. Hence, we obtain a power law

$$\frac{dT}{dz} \propto z^{-2}. \tag{22}$$

Similarly, at the interface between the transition layer and the convection layer, condition (20) yields the following equation:

$$\left(\frac{z}{z_v}\right)^{4/3}\frac{df_v(z/z_v)}{d(z/z_v)} = \left(\frac{z}{z_*}\right)^{4/3}\frac{df_*(z/z_*)}{d(z/z_*)}. \tag{23}$$

We obtain Priestley's similarity law

$$\frac{dT}{dz} = z^{-4/3}. \tag{24}$$

It should be noted that the power law with $\alpha = 2$ does exist between the conduction layer and the convection layer for $Pr \gg Pr_0$ as has been observed by many experiments mentioned before.

(b) *Case* 2, $Pr \ll Pr_0$

The gradient matching condition (20) at the interface between the conduction layer and the transition layer yields

$$\frac{df_c(z/z_c)}{d(z/z_c)} = \frac{df_\kappa(z/z_\kappa)}{d(z/z_\kappa)} = \text{constant} \tag{25}$$

which gives a profile

$$(T_h - T) \propto z \tag{26}$$

where T_h is the temperature of the bottom plate.

A similar application of equation (20) at the interface between the transition layer and the convection layer shows again that Priestley's similarity law (24) must hold in this region.

Now, the linear profile (26) implies that, when $Pr \ll Pr_0$, the conduction layer penetrates deep into the interior fluid to directly contact the convection layer.

The above results are precisely the same as those of Kraichnan's [3] mixing length analysis, which are summarized in Fig. 2. Past objections on the existence of the similarity layer by many experimental studies may be due to the fact that their measurements may not have been made in a range far enough from the

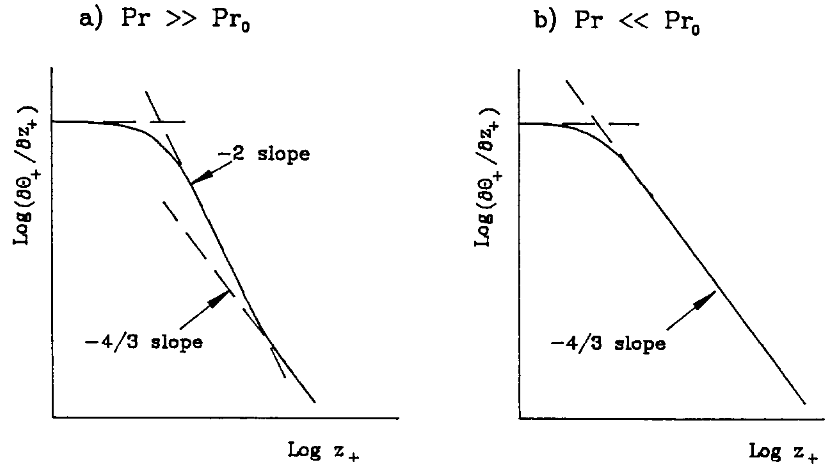

FIG. 2. Theoretical profiles of mean temperature gradients in turbulent thermal convection: (a) $Pr \gg Pr_0$; (b) $Pr \ll Pr_0$.

bottom plate to find the similarity layer, or that the magnitude of Ra is not large enough for the flow field to have the similarity layer (see Long [18]).

4. MEAN TEMPERATURE GRADIENT PROFILE AND ITS WALL-LAYER MODEL

In order to investigate the feasibility to formulate a wall–layer model for the mean temperature gradient profile with the proposed conduction scales, mean temperature profile data reported in various literature are re-analyzed. Although there are a number of experimental results available, many of them could not be used for one of the following reasons: (1) Rayleigh numbers were not large enough to have the similarity layer; (2) they have only a few data points, thus, the mean temperature gradients cannot be obtained; (3) only the mean temperature profiles within a relatively thin layer were available in the reports; (4) necessary experimental values for conversion of the scales are not presented.

Therefore, experimental data of Goldstein and Chu [6] and Yun and Chung [20] for air and Chu and Goldstein [7], Yun and Chung [20] for water are used in the present analysis.

Figure 3 shows the profiles of mean temperature gradients in air for Ra in a range, $8.12\times10^6 \leqslant Ra \leqslant 9.56\times10^7$, presented by Goldstein and Chu [6] and Yun and Chung [20]. The gradient data of the former study were obtained by fitting five successive temperature measurements, using a Mach–Zehnder interferometer, with a second-order polynomial to get the slope at the position of the central point, and those of the latter were calculated by differentiating the cubic spline interpolated curve to the point-wise mean temperature data measured by the resistence wire method. The data points scatter relatively widely. However, it is not difficult to identify the three distinct power law regions; namely, the zero-gradient region in $z_+ < \sim 1.2$, -2 power law region in $\sim 1.2 < z_+ < \sim 12$, and the $-4/3$ similarity region in $z_+ > \sim 15$. Goldstein and Chu [6] claimed that their data fitted to the -2 slope quite well over a large range of z_+ for high Ra. It is, however, interesting to note that their data remarkably follow the $-4/3$ slope in a region $z_+ > 15$, which is wider than that for the -2 slope in a region, $2.5 < z_+ < 8.5$.

Similarly, mean temperature gradient data in water of Chu and Goldstein [7] and Yun and Chung [20] are plotted in Fig. 4. Here, the data for $z_+ > 8$ scatter very widely. However, a close examination of the individual data sets does suggest that there exists a $-4/3$ similarity region again in this water case. It may be argued that Chu and Goldstein [7] have fitted the -3 slope to their data in ref. [7]. However, Fig. 17 in ref. [7] does not seem to support such -3 slope behavior. It is good only for the case of $Ra = 9.34\times10^6$. The

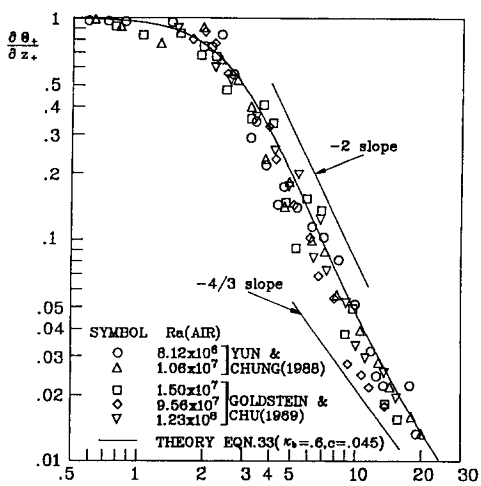

FIG. 3. Mean temperature gradient profiles in Rayleigh convection of air.

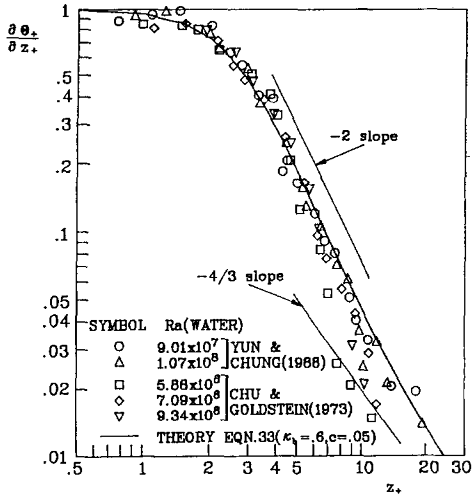

FIG. 4. Mean temperature gradient profiles in Rayleigh convection of water.

other two profiles in the same figure for $Ra = 5.88 \times 10^6$ and 7.09×10^6 vary quite differently from the data of $Ra = 9.34 \times 10^6$, and it is rather a surprise to find that the upper three points of $Ra = 5.88 \times 10^6$ nicely demonstrate the $-4/3$ similarity law, albeit they are low relative to other data.

If the similarity law

$$\frac{d\Theta_+}{dz_+} = \kappa_b z_+^{-4/3} \tag{27}$$

is fitted to the data in Figs. 3 and 4, it can be seen that the 'buoyancy similarity constant' κ_b lies in the range 0.4–0.85, which can be found from the cutting points of the $-4/3$ slope lines through the data to the axis $z_+ = 1.0$.

Now, we are in a position to derive a wall–layer model which represents the thermal structure near the wall region, which can be used as a supplementary aid to the computation of the turbulent thermal convection problem with the current computational models.

The capability to accurately represent the near-wall temperature variation is important in computing the wall bounded flows since intensive variation of the mean temperature profile in the wall layer requires a very large number of computational mesh points. More importantly, an adequate computational model for the near-wall thermal turbulence is not presently available, and, therefore, wall functions are required to provide near-wall boundary values for the turbulence variables under consideration.

As for a reference, formulations of the near-wall velocity profile functions are briefly reviewed as follows. Dean [21] proposed an implicit formula by combining Spalding's [22] implicit function for the mean velocity profile in the viscous sublayer, transition layer and the logarithmic layer with Finley *et al.*'s [23] wake function for the outer layer. In order to obtain a more convenient explicit expression, Musker [24] devised an interpolating formula for turbulent eddy viscosity which is valid both in the near-wall layer and in the logarithmic law of the wall layer. Quite recently, Walker *et al.* [25] and Haritonidis [26] have proposed wall–layer models for the near-wall velocity profile in turbulent flows based on the coherent bursting process in the wall region.

Parallel to the derivation of Musker [24], which is the simplest and easily extendable to the temperature field, a formula for the mean temperature gradient profile in the turbulent thermal convection may be obtained as follows. When an eddy diffusivity model is employed for the turbulent kinematic heat flux, we have

$$\overline{w\theta} \equiv -\kappa_t \frac{\partial T}{\partial z}. \tag{28}$$

Here κ_t is the eddy diffusivity of heat and $\overline{w\theta}$ the turbulent kinematic heat flux in the positive z-direction. Then the non-dimensional energy equation for a horizontal homogeneous field becomes

$$\left(1 + \frac{\kappa_t}{\kappa}\right)\frac{\partial \Theta_+}{\partial z_+} = 1 \tag{29}$$

where the first term on the left-hand side in this equation represents the molecular contribution and the second term, the turbulent eddy contribution to the total heat flux. It may be easily shown that, for a region very close to the wall, the mean temperature profile is very nearly linear

$$\Theta_+ = z_+$$

and that the eddy diffusivity is proportional to the cube of the distance from the wall

$$\frac{\kappa_t}{\kappa} = c z_+^3 \tag{30}$$

where c is a constant to be determined with reference to mean temperature data. This derivation is in exact analogy with that for the turbulent eddy viscosity very near the wall in the turbulent boundary layer or duct flows [24].

In the transition layer, assuming $\kappa_t/\kappa \gg 1$, the power law with $\alpha = 2$ as in equation (22) together with energy equation (29) yields the following approximation:

$$\frac{\kappa_t}{\kappa} \propto z_+^2. \tag{31}$$

Likewise, in the similarity layer, we must have

$$\frac{\kappa_t}{\kappa} = \frac{1}{\kappa_b} z_+^{4/3}. \tag{32}$$

where κ_b is the counter-part of the well-known von Karman constant $\kappa \simeq 0.4$ in the logarithmic velocity profile formula, and may be called the 'buoyancy similarity constant'.

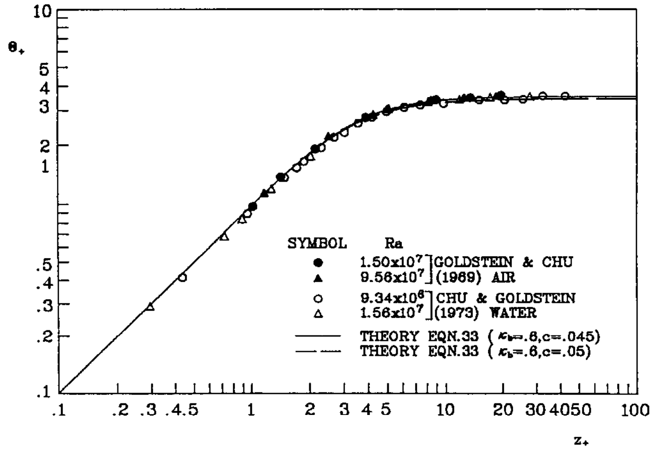

FIG. 5. Mean temperature profiles in Rayleigh convection of air and water.

The variation of κ_t/κ along the vertical distance from the bottom plate may be represented by a form

$$\frac{1}{\kappa_t/\kappa} = \frac{1}{cz_+^3} + \frac{\kappa_b}{z_+^{4/3}} \tag{33}$$

which successively satisfies requirements (30)–(32); i.e. $\kappa_t/\kappa \to z_+^3$ as $z_+ \to 0$, $\kappa_t/\kappa \to z_+^{4/3}$ as $z_+ \to \infty$ and $\kappa_t/\kappa \to z_+^2$ in between these limits.

Substitution of equation (33) into energy equation (29) yields

$$\frac{\partial \Theta_+}{\partial z_+} = \frac{1+\kappa_b c z_+^{5/3}}{1+\kappa_b c z_+^{5/3} + c z_+^3}. \tag{34}$$

The computed profile of the temperature gradient by equation (34) with $\kappa_b = 0.6$, $c = 0.045$ for air and with $\kappa_b = 0.6$, $c = 0.05$ for water are shown in Figs. 3 and 4. The extent of the power law layer of $\alpha = 2$ depends on the constant c, and the temperature gradient for small $z_+ < 10$ is very much more dependent on c than κ_b. The constant c turns out to be a function of Prandtl number, whereas κ_b seems to have a universal value like the von Karman constant $\kappa = 0.4$. However, any conclusive statement cannot be made due to insufficiency of data for different *Pr*.

In computing the turbulent thermal convection, the information about the thermal field required to solve the governing equations for other turbulence quantities, for examples, the turbulent kinetic energy k, the kinematic heat flux $\overline{w\theta}$ and the temperature variance $\overline{\theta^2}$, is the mean temperature gradient, rather than the mean temperature itself. Therefore, the wall–layer model (34) is sufficient as the supplementary relation to turbulence model equations at a certain closure level. However, it may be of practical interest to find the mean temperature profile near the wall region, and it can be easily obtained by integrating the wall function (34) from the wall, $z_+ = 0$. The integrated mean temperature profiles with the constant $\kappa_b = 0.6$ and $c = 0.045$ and 0.05 are compared with the experiments of Goldstein and Chu [6] for air, and of Chu and Goldstein [7] for water in Fig. 5. Overall agreements between the theory and the data are satisfactory, and the comparison clearly demonstrates that the proposed conduction scales perform well to represent the mean temperature profiles near the wall region with a simple wall-layer model.

5. CONCLUSIONS

Under an assumption that a fluid layer in a turbulent thermal convection between two horizontal flat plates has a layered structure, three sets of characteristic scales have been formulated and these are used to confirm the power law behavior of the mean temperature profiles of Kraichnan [3]. The results show that, for high Prandtl number, the fluid layer consists of a conduction layer in which the mean temperature profile is almost linear, a transition layer in which $(dT/dz) \propto z^{-2}$, and a convection layer or a similarity layer in which $(dT/dz) \propto z^{-4/3}$, and that, for low Prandtl number, the fluid layer consists of two layers, a conduction layer and a convection layer.

Experimental mean temperature data available in the literature were collected and re-analyzed with the proposed conduction scales. All of the data used show a good correlation with each other. It is noted that the controversial similarity layer with a temperature gradient of $-4/3$ slope does exist in the interior of the fluid at about $z_+ > 15$.

Finally, based on the conduction scales, a wall–layer model for the mean temperature gradient profile is formulated, which may be used to fill the computational gap between the wall at $z_+ = 0$ and the computational lower boundary point at a certain point off the wall $z_+ \neq 0$ from which computational turbulence model equations at a certain closure level may be integrated.

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ANALYSE D'ECHELLE ET MODELE DE COUCHE PARIETALE POUR LE PROFIL DE TEMPERATURE DANS UNE CONVECTION THERMIQUE TURBULENTE

Résumé—Trois systèmes d'échelles caractéristiques pour la couche de conduction, la couche de transition et la couche de convection sont proposés pour analyser la structure thermique moyenne dans une convection thermique turbulente sans mouvement moyen. Ces échelles sont formulées à partir d'une contribution moléculaire ou turbulente des transports de quantités de mouvement et de chaleur dans chaque couche. En utilisant les échelles proposées et une technique de gradient à l'interface entre deux couches adjacentes, on rétablit la structure multi-couches de Kraichran (*Physics Fluids* **5,** 1374 (1962)) du profil de gradient de température moyenne. Si les échelles de conduction sont utilisées avec les données de gradient de température moyenne non dimensionelle près de la paroi, elles forment une courbe de corrélation plausible qui est presque indépendante des nombres de Prandtl et de Rayleigh pour le domaine des expériences. Par cette courbe, on voit que la couche de convection ou la couche de similitude avec la pente $-4/3$ commence à apparaître après environ $z_+ \sim 15$ et le coefficient de proportionnalité à la loi de puissance $-4/3$ du gradient de température moyenne est d'environ 0,6 soit $d\Theta_+/dz_+ = 0{,}6z_+^{-4/3}$, où Θ_+ et z_+ sont la température et la distance réduites par les échelles respectives de conduction. Ensuite un modèle de couche pariétale pour le profil du gradient de température est formulé en accord avec la loi-puissance, $d\Theta_+/dz_+ \sim z_+^{-\alpha}$ à travers les couches, lequel est en accord convenable avec les données.

ANALYSE DER CHARAKTERISTISCHEN GRÖSSENMASZE UND WANDGRENZSCHICHT-MODELL FÜR DAS TEMPERATURPROFIL BEI TURBULENTER THERMISCHER KONVEKTION

Zusammenfassung—Es werden drei Sätze für die charakteristischen Größenmaße der Wärmeleitungsgrenzschicht, des Übergangsbereichs und der Kovektionsgrenzschicht vorgeschlagen, um die mittlere Temperaturverteilung bei turbulenter thermischer Konvektion zu untersuchen. Diese Größenmaße werden, basierend auf dem Beitrag der molekularen oder turbulenten Wirbel zum Impuls- und Wärmetransport, in jeder Schicht formuliert. Unter Verwendung der vorgeschlagenen Größenmaße und einer Gradientenanpassungs-Technik an der Grenzfläche zwischen zwei benachbarten Schichten wird die mehrschichtige Struktur des Profils des mittleren Temperaturgradienten nach Kraichnan wiederhergestellt (*Physics Fluid* 5, 1374 (1962)). Wenn man die Abmessung der Wärmeleitungsgrenzschicht dazu benutzt, die Werte des mittleren Temperaturgradienten in Wandnähe dimensionslos zu machen, dann ergeben diese eine einleuchtende Korrelationskurve, die im Bereich der experimentellen Daten unabhängig von der Prandtl-Zahl und der Rayleigh-Zahl ist. Anhand der Korrelationskurve zeigt sich, daß die Konvektionsgrenzschicht oder die Ähnlichkeitsgrenzschicht mit der Steigung $-4/3$ bei $z_+ \approx 15$ beginnt und die Proportionalitäts-Konstante des $-4/3$ Potenzgesetzes für den mittleren Temperaturgradienten ungefähr 0,6 beträgt (d.h. $d\Theta_+/dz_+ = 0{,}6z_+^{-4/3}$. Dabei bezeichnen Θ_+ und z_+ die dimensionslose Temperatur bzw. den dimensionslosen Wandabstand unter Verwendung des Größenmaßes der Wärmeleitungsgrenzschicht. Darüber hinaus wird in Übereinstimmung mit dem Potenzgesetz $d\Theta_+/dz_+$ proportional $z_+^{-\alpha}$ ein Wandgrenzschichtmodell für das Profil des mittleren Temperaturgradienten quer durch alle drei Schichten aufgestellt, das die Meßwerte gut wiedergibt.

ИСПОЛЬЗОВАНИЕ АНАЛИЗА МАСШТАБОВ И МОДЕЛИ ПРИСТЕННОГО СЛОЯ ДЛЯ ОПРЕДЕЛЕНИЯ ПРОФИЛЯ ТЕМПЕРАТУР ПРИ ТУРБУЛЕНТНОЙ ТЕПЛОВОЙ КОНВЕКЦИИ

Аннотация—С целью анализа средней тепловой структуры в условиях турбулентной тепловой конвекции при отсутствии среднего движения предложены три формулировки характерных масштабов для проводящего, переходного и конвективного слоев. Указанные выражения формулируются исходя из молекулярного или вихревого вкада в перенос импульса и тепла в каждом слое. С помощью предложенных масштабов и градиентного метода сращивания на границе двух смежных слоев восстанавливается описываемая Крайчнаном (*Physics Fluids* 5, 1374 (1962)) многослойная структура профиля среднетемпературных градиентов. В случае применения масштабов теплопроводности для обезразмеривания данных по среднетемпературному градиенту вблизи стенки получена обобщающая кривая, почти не зависящая от значений чисел Прандтля и Рэлея в исследуемом в эксперименте диапазоне. Из полученной кривой следует, что конвективный или автомодельный слой с наклоном, равным $-4/3$, возникает примерно после $z_+ \sim 15$, а также, что коэффициент пропорциональности степенной зависимости $-4/3$ среднетемпературного градиента составляет примерно 0,6 или $d\Theta_+/dz_+ = 0.6z_+^{-4/3}$, где Θ_+ и z_+-безразмерные температура и расстояние в соответствующих масштабах проводимости. На основе степенной зависимости $d\Theta_+/dz_+ \sim z_+^{-\alpha}$ сформулирована модель пристенного слоя для профиля среднетемпературных градиентов в слоях, которая сорошо согласуется с экспериментальными данными.